Gravitational lensing on the Cosmic Microwave Background by gravity waves

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Abstract

We study the effect of a stochastic background of gravitational waves on the gravitational lensing of the Cosmic Microwave Background (CMB) radiation. It has been shown that matter density inhomogeneities produce a smoothing of the acoustic peaks in the angular power spectrum of the CMB anisotropies. A gravitational wave background gives rise to an additional smoothing of the spectrum. For the most simple case of a gravitational wave background arising during a period of inflation, the effect results to be three to four orders of magnitude smaller than its scalar counterpart, and is thus undetectable. It could play a more relevant role in models where a larger background of gravitational waves is produced.

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The gravitational lensing effect of matter density inhomogeneities on the Cosmic Microwave Background (CMB) radiation has been the subject of several studies [1–8]. It is well known that the deflections undergone by the photons along their path since last scattering can modify the pattern of the observed anisotropies. The effect is to smooth the acoustic or Doppler peaks in the angular spectrum. Although the effect has been found to be small, it should be observable in small angle high accuracy observations [6,8]. It has recently been pointed out [9] that a stochastic background of gravitational waves also contributes to the gravitational lensing of the CMB radiation.

Many scenarios of the early universe may have produced a stochastic background of gravitational waves, as for instance a period of inflation, phase transitions leading to topological defects [10], or bubbles nucleated in a first order phase transition [11]. In many inflationary models, the background of gravitational waves gives a substantial contribution to the CMB anisotropies at large angular scales [12]. These anisotropies arise due to the redshift of the photons induced by the time variation of the graviational waves amplitude along the photon paths.

In this paper we want to quantify the effect on the CMB anisotropies induced by the gravitational lensing of photons from a gravitational wave background. We consider a perturbed flat Robertson–Walker spacetime described in the Poisson gauge by

$$ds^2 = a^2(\eta)\left(-(1+2\varphi)d\eta^2 + \left[(1-2\varphi)\delta_{ij} + \chi^{\top}_{ij}\right]dx^i dx^j\right), \tag{1}$$

where η is the conformal time, in the absence of vector perturbations. φ is the peculiar gravitational potential and $\chi^{\top}_{ij}$ denotes the tensor (transverse and traceless) perturbation.

The gravitational lensing effect on photons is described by the angular displacement $\vec{\beta}$ that measures the difference between the angular direction on the sky from which a given photon arrives to the observer and the one it would have had in the absence of lensing sources along its path. It is given by $\vec{\beta} = r_{\mathcal{E}}^{-1}\mathbf{x}_\perp(\lambda_{\mathcal{E}})$, with $r_{\mathcal{E}} = (\eta_{\mathcal{O}} - \eta_{\mathcal{E}})$ the distance to the last scattering surface and

$$\begin{aligned} x^i_\perp(\lambda_{\mathcal{E}}) &= (\delta^{ij} - e^i e^j)\int_{\lambda_{\mathcal{O}}}^{\lambda_{\mathcal{E}}} d\lambda\left(\chi^{\top}_{jk}e^k - \chi^{\top}_{\mathcal{O}jk}e^k\right) \\ &\quad - (\delta^{ij} - e^i e^j)\int_{\lambda_{\mathcal{O}}}^{\lambda_{\mathcal{E}}} d\lambda(\lambda_{\mathcal{E}} - \lambda)\left(2\varphi_{,j} - \frac{1}{2}\chi^{\top}_{kl,j}e^k e^l\right), \end{aligned} \tag{2}$$

where $\hat{\mathbf{e}}$ is a unit vector denoting the direction of arrival of the photons, and the integration is along the photon background geodesics parametrized by λ. The subscript $\mathcal{O}$ denotes quantities evaluated at the observation point and $\mathcal{E}$ at the emission (or last scattering surface). The term including φ corresponds to the displacement due to scalar density perturbations that has been considered in some previous studies [6–8] and has an observable effect on small angular scales, while the rest describes the effect of the gravitational wave background.

It has been shown by the studies of scalar gravitational lensing that the effect on the CMB anisotropies can be obtained from the autocorrelation function of the transverse displacement

$$\begin{aligned} S(\alpha) &= \langle \beta^j(\hat{\mathbf{e}}_1)\beta_j(\hat{\mathbf{e}}_2)\rangle_{(\hat{\mathbf{e}}_1\cdot\hat{\mathbf{e}}_2 = \cos\alpha)} \\ &= \int\frac{d\Omega_{\hat{\mathbf{e}}_1}}{4\pi}\int\frac{d\Omega_{\hat{\mathbf{e}}_2}}{2\pi}\delta(\hat{\mathbf{e}}_1\cdot\hat{\mathbf{e}}_2 - \cos\alpha)\langle\beta^j(\hat{\mathbf{e}}_1)\beta_j(\hat{\mathbf{e}}_2)\rangle, \end{aligned} \tag{3}$$

where we have taken the mean over all directions separated by an angle α. Once this correlation function is known, we can compute the effect on the temperature correlation function using the methods developed in ref. [5,6,8] for small angular scales, or that in ref. [7] that apply to arbitrary angular scales.

The gravitational wave background in a flat universe can be decomposed as

$$\chi_{ij}^{\top}(\mathbf{x},\eta) = \frac{1}{(2\pi)^3}\int d^3\mathbf{k}\exp(i\mathbf{k}\cdot\mathbf{x})\chi_\sigma(\mathbf{k},\eta)\epsilon_{ij}^{\sigma}(\hat{\mathbf{k}}), \tag{4}$$

where $\epsilon_{ij}^{\sigma}(\hat{\mathbf{k}})$ is the polarization tensor, with σ ranging over the polarization components $+, \times$, and $\chi_\sigma(\mathbf{k},\eta)$ are the corresponding amplitudes. The time evolution of the amplitude during the matter dominated era can be written as

$$\chi_\sigma(\mathbf{k},\eta) \approx A(k)a_\sigma(\mathbf{k})\left(\frac{3j_1(k\eta)}{k\eta}\right), \tag{5}$$

where $a_\sigma(\mathbf{k})$ is a zero mean random variable with autocorrelation function $\langle a_\sigma(\mathbf{k})a_{\sigma'}(\mathbf{k}')\rangle = (2\pi)^3 k^{-3}\delta^3(\mathbf{k}+\mathbf{k}')\delta_{\sigma\sigma'}$, and $j_1(x)$ denotes the spherical Bessel function of first order. The spectrum of the gravitational wave background depends on the processes by which it was generated.

For a wave propagating in the direction $\hat{\mathbf{k}}$, defining a right-handed triad given by $(\hat{\mathbf{k}},\hat{\mathbf{m}},\hat{\mathbf{n}})$, the polarization tensor can be written as

$$\begin{aligned}\epsilon_{ij}^{+}(\hat{\mathbf{k}}) &= m_i m_j - n_i n_j \\ \epsilon_{ij}^{\times}(\hat{\mathbf{k}}) &= m_i n_j + n_i m_j.\end{aligned} \tag{6}$$

In order to compute the autocorrelation function of the angular displacement $S(\alpha)$ induced by the gravitational wave background, we split $x_\perp^i = x_\perp^{\mathrm{I}i} + x_\perp^{\mathrm{II}i}$ with

$$\begin{aligned}x_\perp^{\mathrm{I}i} &= -\frac{1}{2}(\delta^{ij}-e^ie^j)\int_{\lambda_{\mathcal{O}}}^{\lambda_{\mathcal{E}}} d\lambda(\lambda_{\mathcal{E}}-\lambda)\chi_{kl,j}^{\top}e^ke^l, \\ x_\perp^{\mathrm{II}i} &= (\delta^{ij}-e^ie^j)\int_{\lambda_{\mathcal{O}}}^{\lambda_{\mathcal{E}}} d\lambda\left(\chi_{jk}^{\top}e^k - \chi_{\mathcal{O}jk}^{\top}e^k\right).\end{aligned} \tag{7}$$

We thus obtain $S(\alpha) = S^{\mathrm{I}}(\alpha)+S^{\mathrm{II}}(\alpha)$, with $S^{\mathrm{I(II)}}(\alpha) = r_{\mathcal{E}}^{-1}\langle x_\perp^{\mathrm{I(II)}j}x_{\perp j}^{\mathrm{I(II)}}\rangle$, as $\langle x_\perp^{\mathrm{I}j}x_{\perp j}^{\mathrm{II}}\rangle = 0$. The expressions for $S^{\mathrm{I}}(\alpha)$ and $S^{\mathrm{II}}(\alpha)$ can be obtained replacing in eq. (3), the above expressions for $x_\perp^{\mathrm{I}i}$ and $x_\perp^{\mathrm{II}i}$. Parametrizing the photon geodesics by $\lambda \equiv (\eta_{\mathcal{O}}-\eta)/(\eta_{\mathcal{O}}-\eta_{\mathcal{E}})$, so that $\lambda_{\mathcal{O}} = 0$ and $\lambda_{\mathcal{E}} = 1$, and $\mathbf{x} = \hat{\mathbf{e}}\lambda(\eta_{\mathcal{O}}-\eta_{\mathcal{E}})$, we obtain

$$\begin{aligned}S^{\mathrm{I}}(\alpha) = \frac{9}{2}&\int_0^\infty \frac{d\omega}{\omega}\frac{A^2(\omega)}{(2\pi)^3}\int_0^1 d\lambda\int_0^1 d\lambda' j_1(\omega(1-\lambda))j_1(\omega(1-\lambda')) \\ &\times\int_{-1}^{1} d\cos\theta\int_0^\pi d\phi\exp\left(\mathrm{i}\omega\left[\lambda\cos\theta - \lambda'(\cos\theta\cos\alpha - \cos\phi\sin\theta\sin\alpha)\right]\right) \\ &\times(1-\cos^2\theta+\cos\phi\sin\theta\sin\alpha\cos\theta\cos\alpha-\cos^2\phi\sin^2\theta\sin^2\alpha) \\ &\times\Big(2\cos^2\alpha - 1 - 3\cos^2\theta\cos^2\alpha + \cos^4\theta\cos^2\alpha + 2\cos\phi\sin^3\theta\sin\alpha\cos\theta\cos\alpha \\ &+\cos^2\theta + \cos^2\phi\sin^2\alpha(1-\cos^4\theta)\Big),\end{aligned} \tag{8}$$

where we have defined $\omega \equiv k(\eta_{\mathcal{O}} - \eta_{\mathcal{E}})$, and

$$
\begin{aligned}
S^{\rm II}(\alpha) = 18 \int_0^\infty \frac{d\omega}{\omega} \frac{A^2(\omega)}{(2\pi)^3} \int_0^1 d\lambda \int_0^1 d\lambda' \int_{-1}^1 d\cos\theta \left(\exp(\mathrm{i}\omega\lambda\cos\theta) \frac{j_1(\omega(1-\lambda))}{\omega(1-\lambda)} - \frac{j_1(\omega)}{\omega} \right) \\
\times \int_0^\pi d\phi \left(\exp[-\mathrm{i}\omega\lambda'(\cos\theta\cos\alpha - \cos\phi\sin\theta\sin\alpha)] \frac{j_1(\omega(1-\lambda'))}{\omega(1-\lambda')} - \frac{j_1(\omega)}{\omega} \right) \\
\times \Big(\Big[\cos^2\theta - (\cos\theta\cos\alpha - \cos\phi\sin\theta\sin\alpha)^2\Big] \Big[\cos\alpha\sin^2\theta + \cos\phi\sin\theta\sin\alpha\cos\theta\Big] \\
+ \cos\alpha \Big[2\cos^2\alpha - 1 - 3\cos^2\theta\cos^2\alpha + \cos^4\theta\cos^2\alpha \\
+ 2\cos\phi\sin^3\theta\sin\alpha\cos\theta\cos\alpha + \cos^2\theta + \cos^2\phi\sin^2\alpha(1-\cos^4\theta)\Big]\Big), \qquad (9)
\end{aligned}
$$

The integrals over ϕ in the previous equations can be performed analytically in terms of Bessel functions, we do not show the result here. We will instead directly show the result of the numerical integration over the remaining variables for the case of a background of gravitational waves generated during an inflationary period. In this case, the spectrum is nearly scale invariant and proportional to the Hubble constant during inflation. Different inflationary models predict slightly different spectral tensor index n_T and amplitude of tensor modes. For definiteness, we will consider a scale invariant spectrum $n_T = 0$. An upper limit to the amplitude is set by the COBE measurement of CMB anisotropies at large scales. We can thus write the spectrum as $A^2(k)/(2\pi)^3 = 6 \times 10^{-11} T^2(k)$, were we have included the transfer function for gravitons $T^2(k)$ that takes into account the difference in the evolution for modes that entered the horizon during the radiation and matter dominated eras and can be fitted by $T^2(k) = 1 + 1.34(k/k_{eq}) + 2.5(k/k_{eq})^2$ [14], where k_{eq} is the scale that entered the horizon at the equality time.

A useful quantity is the dispersion of the difference of the photons displacement in two directions defined by

$$
\sigma^2(\alpha) = \frac{1}{2} \langle (\beta(\hat{\mathbf{e}}_1) - \beta(\hat{\mathbf{e}}_2))^2 \rangle_{(\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_2 = \cos\alpha)} = S(0) - S(\alpha). \qquad (10)
$$

Figure 1 shows the ratio $\sigma(\alpha)/\alpha$ for a range of angular separations. As the gravitational wave background and the peculiar gravitational potential are two uncorrelated fields, the gravitational lensing displacements that they produce are also uncorrelated. Thus, we can describe the total effect of gravitational lensing as the sum of the scalar and tensor contributions, i. e. $S_{TOT}(\alpha) = S_S(\alpha) + S_T(\alpha)$ and $\sigma^2_{TOT}(\alpha) = \sigma^2_S(\alpha) + \sigma^2_T(\alpha)$, where $S_T(\alpha)$ and $\sigma^2_T(\alpha)$ are the ones computed in this paper and $S_S(\alpha)$ and $\sigma^2_S(\alpha)$ have been computed by several authors [6–8].

It has been shown that the temperature autocorrelation function including the effects of gravitational lensing $\tilde{C}(\theta)$ can be obtained from that in the absence of gravitational lensing $C(\theta)$ if $S(\theta)$ is known. For small angular scales the gravitational lensing effect is to smooth the autocorrelation function as if the smoothing was produced by a Gaussian antenna of width $\sigma(\theta)$.

Comparing the results in Fig. 1 with the corresponding ones for scalar perturbations in refs. [6–8], we see that the dispersion of the graviational lensing displacements induced by gravitational waves is 3 to 4 orders of magnitude smaller than the corresponding scalar

ones. We thus expect that $\sigma_{TOT}(\alpha) \simeq \sigma_S(\alpha)$ and that the effect of the gravitational lensing by the gravitational waves background be undetectable at small scales. This result is essentially due to the fact that the amplitude of the gravitational waves decreases during the matter dominated era for wavelengths smaller than the Hubble radius, and thus their contribution at small scales is supressed. A recent analysis of the effect of a stochastic background of gravitational waves on multiple images and weak gravitational lensing has found a comparable supression with respect to the corresponding scalar perturbations lensing [13].

We could wonder if the very large scale modes, that are the larger amplitude ones, can lead to an observable effect at large angular scales. This is not the case as the effects of gravitational lensing are not evident at scales for which the angular spectrum is smooth. An estimation of this effect using the method proposed in ref. [7] with the correlation $S(\alpha)$ computed in this paper shows that the effect on the temperature anisotropy correlation function at large angular scales can be of the same order of magnitude than that at small angular scales (on the contrary, for scalar perturbations the effect at large angular scales is much smaller than the small scales one). This is however too small to be detectable, besides the fact that cosmic variance at large angular scales make small variations in the predicted spectrum untestable.

The results discussed above have been obtained for a gravitational wave background produced during a period of inflation. There are however other scenarios of the early universe in which a larger background of gravitational waves is expected. This is the case for example for models with cosmic strings [10] or first order phase transitions [11]. The method developed in this paper can be applied to any of these cases just by replacing the corresponding spectra $A(\omega)$ in eqs. (8) and (9). It should be taken into account that also the temperature anisotropies and the scalar gravitational lensing dispersion may be different in alternative theories.

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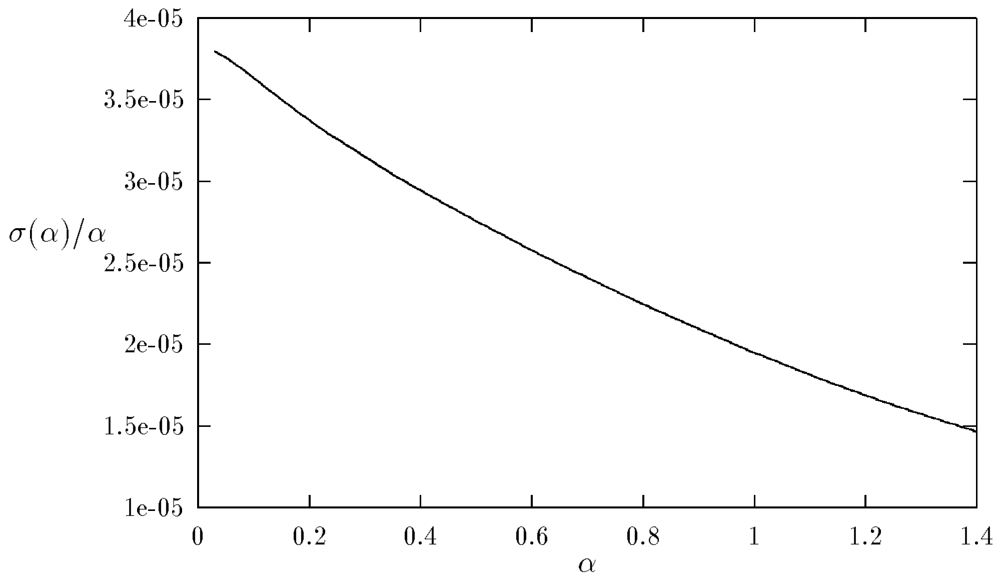
Figure 1: $\sigma(\alpha)/\alpha$ vs. α for a range of α in units of radians.



4e-05
3.5e-05
3e-05
2.5e-05
2e-05
1.5e-05
1e-05
$\sigma(\alpha)/\alpha$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
α